



KTH Engineering Sciences

2008-09-01

SF2723 Topics in Mathematics III – Matrix Groups

Course description

There are several different sets of matrices occurring naturally in different areas of mathematics that all have the property that they are closed under multiplication and taking inverses. Some of these are called *the classical groups* and you have certainly already met them in some form, even if you don't know them by name. They are the *general linear group*, GL_n , the *special linear group*, SL_n , the *orthogonal group*, O_n , the *special orthogonal group*, SO_n and the *symplectic group*, Sp_n .

During this course, we will go through several important concepts from the 19th and 20th century mathematics which will help us in understanding what the classical groups are like. We will use algebraic, topological, differential geometric and algebraic geometric methods. The idea is that the matrix groups, which themselves have important roles in mathematics and physics, will serve as tools for understanding the abstract concepts needed in order to study them.

SF2723

Credits: 7,5

Level: advanced level

Grading: A-F

Language: English

Responsible teacher

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Examination

In order to pass the course one has to either take the written final exam, which will take place on December 17, 8.00-13.00, or deliver solutions to the homework problems during the course and take a 30 minutes oral exam at the end of the course. The homework problems will be distributed after each lecture and solutions should be handed in within a week. The total maximal score on the homework assignments is 200 and in order to pass, at least 100 is required.

Time budget

The 7,5 credit course corresponds to a total of approximately 160 hours of course work which means that the students are supposed to spend ten hours per week on the course during the whole semester. Two of these hours correspond to the lectures and the remaining eight hours should be used for reading the literature, working on exercise problems and solving homework problems.

Schedule

<i>Date</i>	<i>Time</i>	<i>Content</i>
3/9	15.15-17.00	<ul style="list-style-type: none"> • 1.1 Matrix groups • 1.2 Groups • 1.3 Rings and fields
10/9	15.15-17.00	<ul style="list-style-type: none"> • 1.4 Matrix groups over arbitrary fields • 1.5 Generators for groups • 1.6 Vector spaces
17/9	15.15-17.00	<ul style="list-style-type: none"> • 1.7 Bilinear forms • 1.8 The orthogonal and symplectic groups • 1.9 Generators of the orthogonal and symplectic groups • 1.10 The center of the matrix groups
24/9	15.15-17.00	<ul style="list-style-type: none"> • 2.1 Norms and metrics on matrix groups • 2.2 The exponential map • 2.3 Diagonalization of matrices and the exponential and logarithmic functions
1/10	15.15-17.00	<ul style="list-style-type: none"> • 2.4 Analytic functions • 2.5 Tangent spaces of matrix groups • 2.6 Lie algebras of the matrix groups • 2.7 One parameter subgroups of matrix groups
8/10	15.15-17.00	<ul style="list-style-type: none"> • 3.1 The Inverse Function Theorem • 3.2 Matrix groups in affine space
15/10	15.15-17.00	<ul style="list-style-type: none"> • 3.3 Topological spaces • 3.4 Manifolds • 3.5 Equivalence relations and applications
22/10		B R E A K
29/10	15.15-17.00	<ul style="list-style-type: none"> • 3.6 Tangent spaces • 3.7 The tangent space of zeroes of analytic functions • 3.8 Connectedness • 3.9 Compact topological spaces
5/11	15.15-17.00	<ul style="list-style-type: none"> • 4.1 Lie groups • 4.2 Lie algebras • 4.3 Vector fields • 4.4 The Lie algebra of a Lie group
12/11	15.15-17.00	<ul style="list-style-type: none"> • 4.5 One parameter subgroups of Lie groups • 4.6 The exponential function for Lie groups
19/11	15.15-17.00	<ul style="list-style-type: none"> • 5.1 Affine varieties • 5.2 Irreducibility of the matrix groups • 5.3 Regular functions
26/11	15.15-17.00	<ul style="list-style-type: none"> • 5.4 The Hilbert Nullstellensatz
3/12	15.15-17.00	<ul style="list-style-type: none"> • 5.5 Prevarieties • 5.6 Subvarieties
10/12	15.15-17.00	<ul style="list-style-type: none"> • 5.7 The tangent space of prevarieties • 5.8 Tangent spaces for zeroes of polynomials
17/12	08.10-13.00	Final written exam.